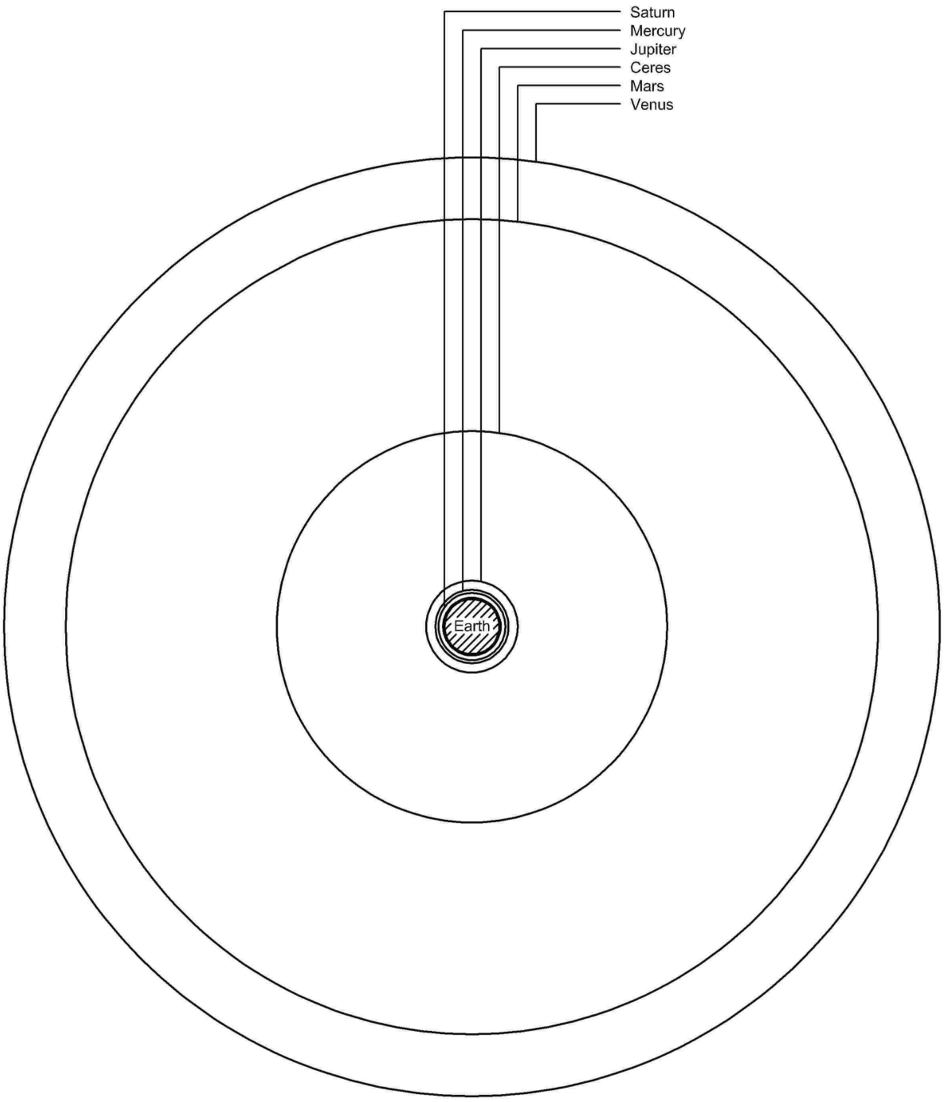
INTERPLANETARY GATE ORBITS

Cost-effective Circular Orbits for Transfer to Other Planets



by

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ABBREVIATIONS USED

IGO	Interplanetary Gate Orbit
MGO	Mars Gate Orbit (around earth)
EGO	Earth Gate Orbit (around Mars)
LEO	Low Earth Orbit
HEO	High Earth Orbit
IOT	Interorbital Tug
OTV	Orbital Transfer Vehicle
ITV	Interplanetary Transfer Vehicle
ISP	Specific Impulse

ABSTRACT

At certain specific circular orbits around a planet, the additional delta-v required to go from orbital to a given hyperbolic escape velocity is minimal. The effectiveness in terms of cost/unit mass sent on an interplanetary trip, of launching from these specific orbits, with regards to launching from Low Earth Orbit is shown.

1. Introduction

The velocity needed to escape a planet's gravitational attraction decreases as distance from the planet increases. Similarly, circular orbital velocity also decreases as distance from the orbited body increases. At any given circular orbit, any additional velocity given to the orbiting body will cause its injection into an elliptical orbit with the point of acceleration at its periapsis. If the additional velocity is sufficient, the orbiting body will escape the central body's attraction entirely following a parabolic (ellipse with eccentricity = 1) or hyperbolic path (ellipse with eccentricity > 1).

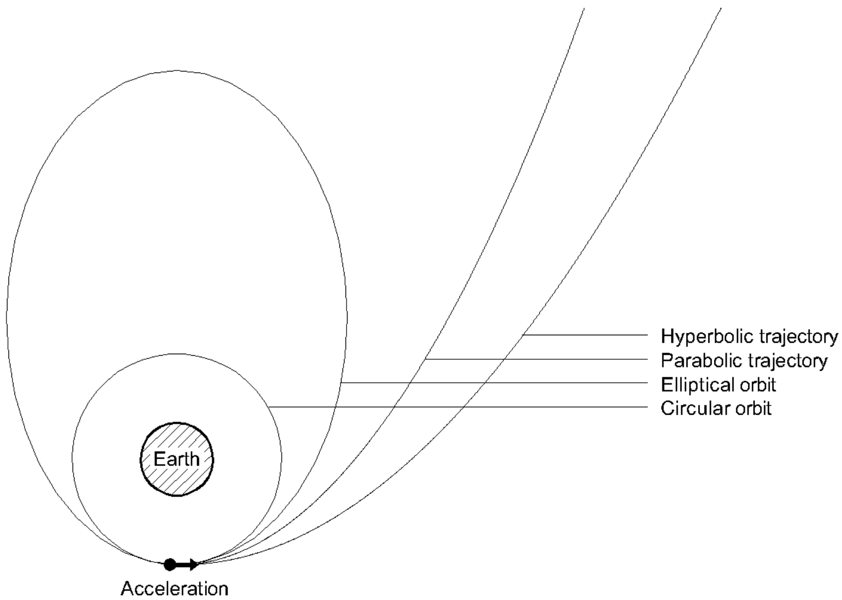

Figure 1. Orbits and tra=ectories

In order to leave an Earth orbit for another planet, there is a minimum velocity required to firstly, escape the Earth's gravity, and secondly, engage on a heliocentric transfer trajectory that will intersect the orbit of the destination planet. The minimum delta-v required is that necessary for a Hohmann transfer, i.e. a heliocentric elliptical orbit with departure planet at periapsis and destination planet at apoapsis. At departure and arrival, each planet is on opposite sides of the sun.

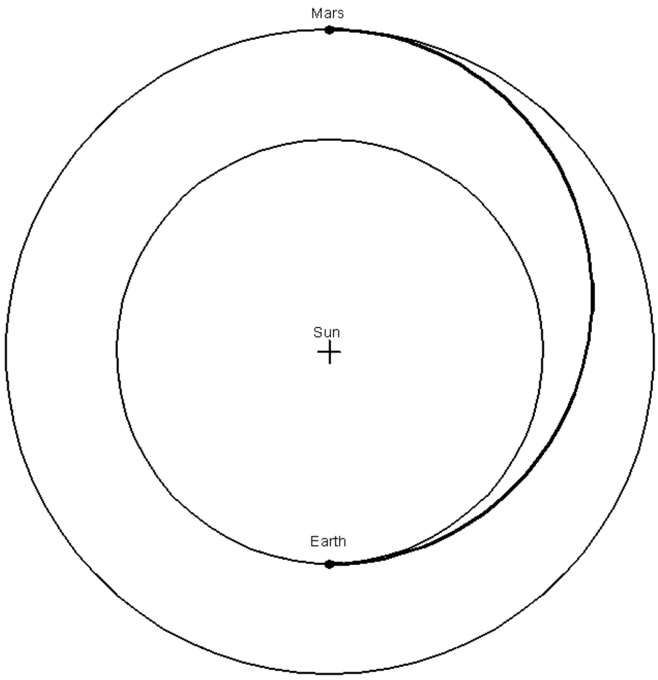

Figure 2. Hohmann transfer between the Earth and Mars (simplification)

Delta-v required to initiate interplanetary Hohmann transfer seems to decrease with distance from the planet center. However, there is a specific distance from the center of each planet where the delta-v required for Hohmann transfer to any other planet is lowest. Above and below this orbit, required delta-vs are higher for interplanetary transfer.

This paper proposes to name these specific orbits Interplanetary Gate Orbits, because they are the gates to cheap access to the other planets.

First the locations of these orbits around the Earth will be defined, and their unique astrodynamic properties demonstrated. Next, a case study, featuring a hypothetical Mission to Mars, will demonstrate their economical value.

2. Interplanetary Gate orbits

Orbital velocity for a circular orbit around the Earth is given by:

$$v_o = \sqrt{(\mu/r_o)} \quad (1)$$

Where v_o = orbital velocity in km/s, μ = the standard gravitational parameter for Earth (398'600 km^3/s^2) and r_o = orbital radius in km.

At this distance from Earth, the velocity required to inject a vehicle into an escape hyperbola with a semi-major axis "a" is given by the Hyperbola Equation:

$$v_h = \sqrt{(2\mu(1/r_o+1/2a))} \quad (2)$$

Where v_h = hyperbolic velocity at periapsis in km/s, μ = standard gravitational parameter for Earth (398'600 km^3/s^2), r_o = orbital radius in km , and a = semi-major axis of hyperbolic escape trajectory.

Note that at r_o = infinity, equation (2) is reduced to:

$$v_h = \sqrt{(\mu/a)} \quad (3)$$

From this we derive the so-called C3, or Characteristic Energy of the escape trajectory, which is the square of the escape velocity, expressed in km^2/s^2:

$$C3 = \mu/a \quad (4)$$

To obtain the semi-major axis of the escape hyperbola, we invert equation (3) or (4), replacing v_e by the required velocity for Hohmann transfer (2.94 km/s) or C3 by required Characteristic Energy (8.64 km^2/s^2):

$$a = \mu/ v_h^2 \quad \text{or} \quad a = \mu/C3 \quad (5)$$

If the vehicle is already in a circular orbit at the hyperbola's periapsis, the net additional velocity required to inject into the hyperbolic trajectory with velocity V_h is:

$$\mathbf{v_e = v_h - v_o} \quad \mathbf{(6)}$$

Combining equations (1) and (2) to obtain the net delta-v required to leave a circular orbit for a Hohmann transfer to Mars:

$$\mathbf{v_e = \sqrt{(2\mu(1/r_o+1/2a)} - \sqrt{(\mu/r_o)}} \quad \mathbf{(5)}$$

Comparing V_h and V_o on a graph we notice a point of minimum divergence, where V_e is therefore at a low point:

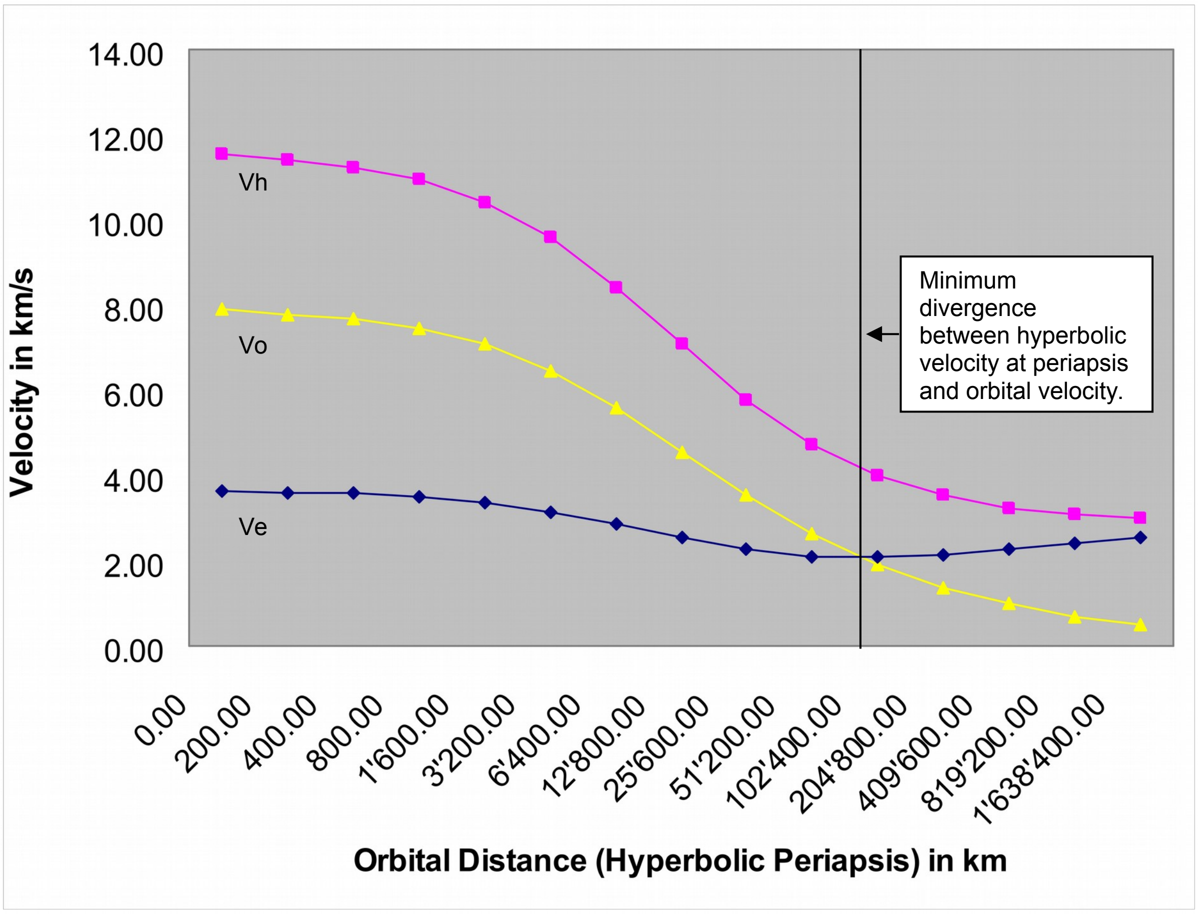

Figure 3.Comparing orbital velocity, hyperbolic velocity and additional velocity required for hyperbolic escape.

This minimal divergence is due to the fact that V_h is proportional to the square root of (1/r + a constant) whereas V_o is proportional only to the square root of (1/r). Differentiation shows that the slope of V_o exceeds that of V_h when r > 92’042 km. The net effect of this is that V_e has a minimum value at r = 92’042 km. In other words, a minimum delta-v for Hohmann transfer to Mars exists at an orbital radius of approximately 92’042 km.

Table 1 shows a series of orbital and hyperbolic velocities around the Earth. You will note that orbits above and below the Mars Gate Orbit (MGO) have higher v_e requirements for Mars transfer.

Table 1: Characteristics of several circular orbits around the Earth

Orbit height (km above surface)	Total velocity for Hohmann transfer to Mars (km/s)	Orbital velocity (km/s)	Net delta-v required for Mars transfer (km/s)
0.00	11.57	7.91	3.66
200.00	11.40	7.79	3.61
400.00	11.24	7.67	3.57
1'000.00	10.81	7.35	3.45
2'000.00	10.19	6.90	3.29
4'000.00	9.25	6.20	3.05
8'000.00	8.01	5.27	2.74
16'000.00	6.66	4.22	2.43
32'000.00	5.43	3.22	2.20
64'000.00	4.47	2.38	2.09
92'042.15	4.09	2.01	2.08 (MGO)
128'000.00	3.82	1.72	2.10
256'000.00	3.42	1.23	2.19
512'000.00	3.19	0.88	2.32
1'024'000.00	3.07	0.62	2.45
2'048'000.00	3.01	0.44	2.57

The complete list of Earth Gate Orbits for each planet of the solar system is shown in table 2:

Table 2: Earth Interplanetary Gate Orbits

Destination planet	Required delta-v for Hohmann (km/s)	Gate Orbit altitude (km from surface)	Gate Orbit (km from center)
Mercury	9.83	1'879	8'250
Venus	2.74	99'815	106'186
Mars	2.94	85'859	92'230
Ceres	4.24	37'973	44'344
Jupiter	8.77	3'994	10'365
Saturn	10.28	1'173	7'544
Uranus	11.25	-72	6'299
Neptune	11.63	-477	5'894
Pluto	11.78	-626	5'745

As can be seen, the higher the delta-v required for a given interplanetary transfer, the lower the radius of the Gate Orbit around Earth.

3. Mathematical Proof

At the point where V_e is lowest on the graph, the derivative of V_e equals zero or, in other words, from equation (6), the derivatives of V_h and V_o are equal to each other:

$$\mathrm{Dx}[V_o] = \mathrm{Dx}[V_h] \tag{6}$$

Deriving equations (1) and (2):

$$\mathrm{Dx}[V_o] = \sqrt{(\mu/4r_o^3)} = \mathrm{Dx}[V_h] = \sqrt{(\mu a/r_o^3(2a+ r_o))} \tag{7}$$

Squaring both sides and eliminating redundant variables on each side gives:

$$1/4 = a/(2a+ r_o) \tag{8}$$

Simplifying gives:

$$r_o = 2a \tag{9}$$

Replacing a by C3 from equation (4):

$$\boxed{r_o = 2\mu/C3} \tag{10}$$

Thus, it has been shown that the Interplanetary Gate Orbit radius for a given C3 and gravitational parameter is equal to two times the gravitational parameter divided by C3.

Now, to determine additional velocity required at the Gate Orbit to inject into an interplanetary trajectory, combining equations (5) and (9):

$$v_e = \sqrt{(2\mu(1/2a +1/2a)} - \sqrt{(\mu/2a)} \tag{11}$$

$$v_e = 2\sqrt{(\mu/2a)} - \sqrt{(\mu/2a)} \tag{12}$$

$$v_e = \sqrt{(\mu/2a)} \tag{13}$$

Or, to express in terms of C3, using equation (4):

$$\boxed{v_e = \sqrt{(C3/2)}} \tag{14}$$

Thus, it has been shown that the additional velocity required to leave a Gate Orbit for a given C3 and gravitational parameter is equal to the square root of C3 divided by 2.

For example, the Mars Gate Orbit can be calculated at ((2x398'600/2.943^2=) 92'042 km radius. When in circular orbit at this distance from Earth, a vehicle will require an additional velocity of (√(2.943^2/2)=) 2.08 km/s to inject into a Hohmann transfer to Mars.

4. Strategic importance

For any given interplanetary transfer there exists a circular orbit with the lowest possible escape delta-v requirement. For vehicles departing to or arriving from a given destination in the solar system, there is therefore an optimal arrival/departure orbit where the delta-v requirement is lowest. The radius of this circular orbit is referred to by the term Gate Orbit and can be calculated using equation 14.

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